In measuring the wide bands recorded in the above lists the middle position of the bands has been taken. There is evidence that some of these bands could be resolved into series by using a different type of spectrograph.

DESCRIPTION OF PLATES.

PLATE 13.

Fig. 2.—Spectrum of 1 mgrm. of sulphur.
Fig. 3.—Spectrum of 8 mgrm. of sulphur.

PLATE 14.

Fig. 5.—Spectrum of sulphur vapour at constant pressure of 768 mm. Tube 100 mm. long.

An Apparatus for the Direct Determination of Accelerations.

By the late Prince B. GALITZIN, For. Mem. R.S.*

The question of the determination of the acceleration of the true motion of the ground in various seismic phenomena, or of the motion in different parts of buildings, bridges, and all kinds of artificial structures, due to explosions, shocks, or oscillations of the ground, has a considerable theoretical and practical interest, since the investigation of these accelerations serves as a guide in the study of the mechanical forces by which these movements are caused. The knowledge of these forces is particularly important in the design of buildings of all kinds in seismological areas, and also for the calculation of various elements (ties or reinforcements) of structures which are often subjected to vibrations caused by the action of powerful engines by shifting of large masses (in the case of bridges or buildings on yielding foundations), by sudden shocks (in the case of ships armed with heavy guns), and so on.

The problem of the determination of the true magnitude of the accelera-

* The two papers, by Prince Galitzin, which follow, are translated from the originals, which appeared in the 'Bulletin of the Academy of Science of Petrograd' for 1915. Their publication in the 'Proceedings' has, for special reasons, been authorised by the Council of the Royal Society.

Prince Galitzin was elected a Foreign Member of the Royal Society on March 23, 1916, and died May 17 of the same year. The papers contain, therefore, the last work of a distinguished man of science. It was represented to the Council that they were of high scientific value, but inaccessible to most men of science, owing to their only having appeared in the Russian language.

Under these circumstances the Council felt justified in departing from the usual practice of not publishing communications that have already appeared elsewhere.—A.S.

tion, however, presents great practical difficulties, especially in the case where the amplitudes of the motion are small and the oscillations of short period. The apparatus generally used for this purpose consists mainly of a pendulum—whether simple, horizontal, or elastic—according to the anticipated magnitude of the period and of the three components (two horizontal and one vertical) of the movement under investigation.

In what follows we shall consider only the horizontal movements; the same arguments, however, may be applied to the case of vertical movements.

Let us suppose a small horizontal platform to be moving parallel to the axis of X, and let it be required to determine the acceleration of this motion. On this platform an apparatus is mounted which can oscillate parallel to the same axis with a natural period, T. The movements of this apparatus are recorded by a revolving registering mechanism, which participates in the movement of the platform. Let y be the displacement (relative to the registering mechanism) of the recording part of the apparatus from its position of equilibrium, at the instant t, and x the displacement at the same instant of the platform, its normal position. Since x is usually extremely small, the apparatus used for its investigation must be very sensitive.

In the general case x is an arbitrary and unknown function of t, say

$$x = f(t). \tag{1}$$

In this case, as is well known, the differential equation of the motion of the apparatus may be expressed in the following standard form:

$$y'' + 2\epsilon y' + n^2 y + \sigma x'' = 0, \tag{2}$$

where ϵ is the constant of damping of the apparatus, σ is a constant depending on the sensitiveness, and

$$n = \frac{2\pi}{\mathrm{T}}.$$

If in the registering apparatus the time is also recorded, the record will always represent y as a function of t, say

$$y = \mathrm{F}(t).$$

F (t) is thus always known, at least numerically. In this case the acceleration x'' may be determined directly from formula (2), if the constants ϵ, σ, and n are known; thus

$$x'' = -\frac{1}{\sigma}[\mathrm{F}''(t) + 2\epsilon \mathrm{F}'(t) + n^2 \mathrm{F}(t)]. \tag{3}$$

This simple theoretical solution of our problem, however, has no practical value, since the differentiation of the empirical function $y = \mathrm{F}(t)$ is usually impossible. In the case, however, where the motion of the platform, x, may

be considered as simple harmonic, or as a resultant of several simple sinusoidal oscillations, the problem may be easily solved. Suppose

$$x = a \sin pt, \tag{4}$$

where a is the amplitude and

$$T_p = \frac{2\pi}{p} \tag{5}$$

is the period of the oscillations.

Integrating on this supposition the equation (2), it may easily be shown* that the motion of the apparatus gives a more complicated curve, consisting of two parts: one part, corresponding to the motion of the apparatus itself, represents a damped sinusoid with a period $T' = 2\pi/\sqrt{(n^2 - \epsilon^2)}$; while the second part, corresponding to the motion of the platform, consists of harmonic oscillations with a period T_p. The period T' of the motion of the apparatus itself always interferes with the records, and makes it difficult to determine the amplitudes and the periods of the movement of the ground. These difficulties, however, may be overcome in various ways. One is to increase as much as possible the damping of the apparatus, in which case the movements due to the apparatus itself die quickly away, so that only the movements of the platform are recorded. This is the method generally used in the Russian seismological stations. Since the sensitiveness of the apparatus diminishes considerably with increase of damping, very sensitive registering methods must be used in this case.

In the case of oscillations with small periods, another method of determining the period T_p of the platform may be used. The damping of the apparatus is now made very small so as not to reduce the sensitiveness; on the other hand, the dimensions and the shape of the apparatus are chosen in such a way as to render the period of its own motion large compared with T_p. The oscillations of the platform will then easily be distinguished from the large waves caused by the motion of the apparatus itself, so that a detailed study of the records may lead to a more or less accurate determination of the period T_p.

By either of these two methods the amplitude a and the period T_p may be determined, and then the acceleration x'' in the case of simple harmonic oscillations may be calculated from formula (4), namely:

$$x'' = -p^2 a \sin pt. \tag{6}$$

The maximum value of the acceleration will be

$$w_m = \left(\frac{2\pi}{T_p}\right)^2 a. \tag{7}$$

* See my 'Lectures on Seismology,' chap. v, § 3.

We thus see that in the case of simple harmonic oscillations the acceleration may be determined by means of some type of pendulum. In the case, however, where the motion under consideration is the resultant of several sinusoidal oscillations, the determination of the acceleration becomes more difficult. If even we succeed in eliminating the movement due to the apparatus itself, the results given by the records still remain very complicated. From these results we must find out the amplitudes and periods of the components forming the motion under consideration, and this problem, if not impossible, is usually exceedingly troublesome. In the case where x is an arbitrary function of t, the methods mentioned above fail to give us any reliable information about a and T_p.

Nevertheless, a knowledge merely of the maximum value of the acceleration is of immense practical importance, and it would be very useful to possess an apparatus which would give the instantaneous value of the acceleration due to any arbitrary function $x = f(t)$.

It will be shown in what follows that such an apparatus may be constructed. Fig. 1 shows the principle of this apparatus. A solid cast-iron platform, BD, is rigidly connected to a prismatic cast-iron body, ABC, having in section the form of a right-angled triangle. This body has on its upper portion a projection from which at E is suspended, by means of a rigid bar, EG, a heavy mass, M, which can move only in the plane of the figure. To this mass is attached a small pin b touching without pressure the plate s;

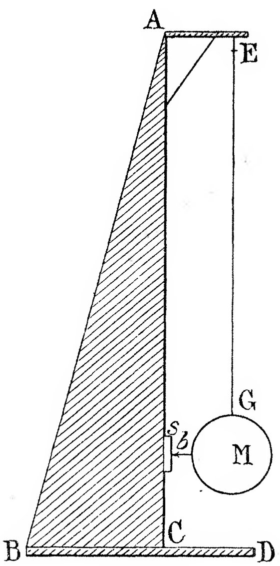

Fig. 1.

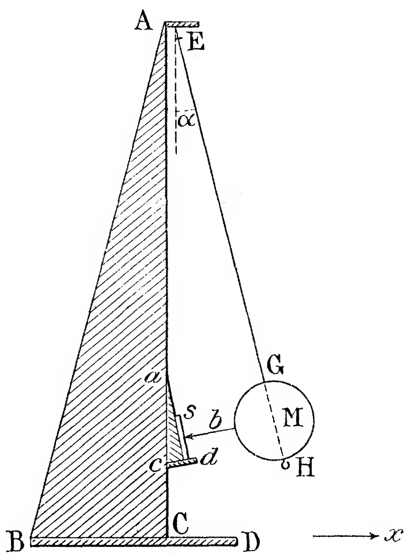

Fig. 2.

the direction of this pin passes through the centre of gravity of the whole suspended system. In the normal position of the apparatus the pressure of the pin on the plate s is zero, but when it is moving in the positive direction of the axis of X with an acceleration x'' the pressure on the plate is Mx''. It is evident that when the apparatus moves in the opposite direction, the rod leaves the plate owing to the inertia of the mass M. In order to avoid this, the apparatus must be slightly altered, as shown in fig. 2.

At the right side of the frame, ABC, a small projection acd is made, on which the plate s is fixed at an angle α with the vertical. The length of the pin b is such that the suspended mass M forms the same angle α with the vertical line. The normal pressure of the mass M on the plate s in this apparatus is

$$P_0 = Mg \sin \alpha ; \tag{8}$$

when the apparatus is moving with an acceleration x'', the pressure is

$$P = Mg \sin \alpha + Mx'' \cos \alpha. \tag{9}$$

In order that the rod may always remain in contact with the plate s it is necessary to choose the angle α so as to satisfy the expression

$$tg\alpha > \frac{w_m}{g},$$

where Wm is the greatest absolute value of the acceleration which may be met with. Since g is comparatively very large, the angle, α, is usually small.

Let $p = P - P_0$; then putting w instead of x'', we get

$$p = Mw \cos \alpha. \tag{10}$$

Thus the pressure p is always proportional to the acceleration, quite independently of the form of the function $x = f(t)$.

The question discussed above is thus reduced to that of finding an experimental arrangement which would make it possible to determine directly the momentary value of p.

The idea of using for this purpose any kind of elastic plates, membranes, springs, or dynamometers must be entirely excluded, since any such arrangement is always accompanied by a displacement of the point of application of the force, and inevitably introduces into the system new oscillations with a period of their own; we thus make no further step towards the solution of the problem. For the determination of p an arrangement must be devised which is not subject to any appreciable displacement and does not introduce a new period of oscillation.

Amongst the various physical phenomena there is one which completely satisfies these conditions. It is the phenomenon of piezo-electricity.

If we place a plate of quartz or tourmaline between two metallic sheets

and apply a pressure to it, a free electric charge appears on the metal sheets, which is within large limits proportional to the pressure. We shall see later how, by means of the apparatus shown in fig. 2, we can determine the acceleration by measuring this charge.

The most suitable instrument for measuring the charge is Luts-Edelmann's string electrometer. This consists of a very thin metal thread a few microns in diameter fixed in a vertical position between two plates charged to the potentials $+v$ and $-v$. When the string receives a charge it bends towards one of the plates in accordance with the sign of the charge. The displacement of the central portion of the string is within large limits proportional to the charge, but even if it is not so it is always easy to make a table for the corresponding corrections. By means of a microscope provided with a micrometric scale, the displacement of the string, and consequently the charge received by it, may be easily measured. The sensitiveness of the electrometer may be regulated by varying the tension of the string, or the potential of the plates; the sensitiveness, however, must not be made too great, otherwise the zero position of the apparatus becomes unstable. The apparatus responds rapidly to any charge of voltage on the string; it has a small capacity and does not introduce any period of oscillation of its own. The string and the plates of the apparatus must be well insulated, and the air within carefully dried; the walls of the apparatus are earthed, and the apparatus itself placed in an uninsulated cage in order to screen it from external electrical disturbances.

When the string of the electrometer moves it is easy to obtain an automatic record of the movement of the central portion of the string. For this purpose it is necessary to remove the eyepiece of the microscope and to place the objective sufficiently near to the string, so that at a distance of about 3 metres the image of the string is formed on the surface of a revolving drum with a horizontal axis, covered with photographic paper. The string is illuminated from the opposite side by a parallel beam of light from a Nernst lamp, and in front of the revolving drum itself is placed a cylindrical lens concentrating the rays in a vertical plane. By means of a diaphragm placed in front of the revolving drum, the image of the central part of the string may be formed on the photographic paper as a dark point on an illuminated background. In this way we get, after developing the photographic paper, a white line on a black ground. It is advisable to use in these experiments a special electromagnetic device in contact with a clock in order automatically to mark the time on the records. The photograph will then represent the movement of the string of the electrometer in the smallest details.

Instead of placing the registering apparatus at a considerable distance from the electrometer in order to obtain sufficient magnification, it may be convenient, in some cases, to make use of microphotography and to record the movement of the string on a winding photographic film. It is then possible to mount the electrometer and the registering instrument on the same foundation, which makes the whole apparatus very compact and portable.

Let us now return to the apparatus described above (fig. 2). A quartz plate inserted between two metal sheets is fixed in the position of *s*. The sheet at the back (in contact with the support ABC), as well as the whole apparatus, is earthed, while the front sheet covering the quartz plate is connected by a thin well-insulated wire to the string of the electrometer. In order to insulate this sheet from the pin *b* an ebonite plate is inserted between them. It is advisable to enclose the system of plates and sheets in a glass vessel containing metallic sodium, a small opening being left in the front surface of the cylinder, through which the pin may freely move. The whole apparatus, as well as the wire leading to the electrometer, is surrounded by an uninsulated metal cage.

If the dimensions of the apparatus are sufficiently small, it may be mounted on the same platform with the electrometer and the registering apparatus. In this case the whole apparatus for the determination of acceleration may be easily removed from one place to another.

When the pin *b* is first placed in contact with the ebonite plate, a charge appears on the string of the electrometer caused by the normal pressure P_0 (see formula 8). This charge must be removed by earthing the string, whereby the zero line on the registering drum is determined. When this charge is large care must be taken not to break the string by a too rapid discharge when putting it to earth.

The connection of the string with the earth is then broken, after which the apparatus shows only the variations of pressure $p = P - P_0$. The deviation of the image of the string from the zero position caused by this pressure, is proportional to p, and we may therefore put

$$y = kp. \tag{11}$$

Comparing this formula with formula (10) for the determination of the acceleration w, we get finally,

$$w = Ay, \tag{12}$$

where

$$A = \frac{1}{kM\cos\alpha}. \tag{13}$$

A is a constant depending on the sensitiveness of the apparatus, which can be easily determined.

Formula (12) shows that the acceleration is proportional to the ordinate y of the curve traced on the revolving drum owing to the displacement of the string, the sign of y changing with that of w; consequently, knowing A we can always determine the acceleration w. As shown above, the apparatus gives without lag the value of w for any arbitrary movement of the platform. Thus the problem discussed in the beginning of this paper may be considered to be completely solved.

The constant, A, of the apparatus is easy to determine. Since M and α are known, it remains only to find the value of the coefficient k, which depends on the properties of the apparatus itself and of the registering device. For this purpose the string of the electrometer is first earthed, while the registering instrument is allowed to work for a short time, in which case the record gives the zero line. The string is then disconnected from the earth, and a small weight m is added to the mass M by means of the hook H (fig. 2).

Let L be the distance of the hook H from the point E and l the distance from the same point of the centre of gravity of the mass M. It is easy to see then that the weight m will produce on the surface of the quartz plate an additional pressure p_1, where

$$p_1 = mg \sin \alpha \frac{\mathrm{L}}{l}. \tag{14}$$

This pressure will cause a deflection y_1 of the string of the electrometer. Taking into consideration formula (11), we find

$$\mathrm{K} = \frac{l}{\mathrm{L}} \cdot \frac{y_1}{mg \sin \alpha}. \tag{15}$$

The value of K thus determined may be checked in the following way. After having obtained the record of the deflection of the string produced by the pressure p_1 the string is again earthed, which causes the illuminated spot on the revolving drum to go back to the zero line. The small weight is then removed from the hook, whereby the string receives a deflection in the opposite direction. If the normal position of the string of the electrometer be half-way between the plates, these two deflections are equal in magnitude. In the case of a small difference between them the mean value of the two deflections may be taken. When fixing or removing the weight m, care must be taken not to introduce any additional accidental changes in the electrometer by shaking the apparatus.

Introducing the value of K from formula (15) in the formula (13), we get finally:

$$\mathrm{A} = \frac{\mathrm{L}}{l} \cdot \frac{m}{\mathrm{M}} \cdot \frac{g}{y_1} \cdot tg\alpha. \tag{16}$$

The determination of A is thus very easy. It should be determined always before proceeding to observations.

In the case where the acceleration w is known, the constant A may be calculated directly from formula (12). Suppose that a platform, suspended by four bars attached to the corners, executes regular harmonic oscillations backwards and forwards, remaining at the same time in a horizontal plane. If a purely mechanical record of the movement of this platform is made on a registering drum, the amplitude and the period of the oscillations of the platform may be determined, and thence the acceleration w is deduced. Such a platform was constructed at the physical laboratory of the Academy of Science (Petrograd), and has proved to be of great practical use in testing various kinds of apparatus.

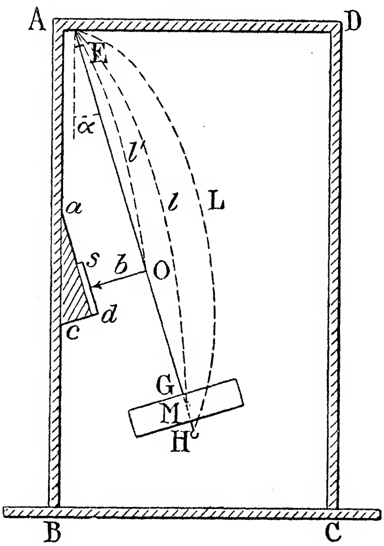

Fig. 3.

The theory of the new apparatus for the determination of accelerations has been described above. Before constructing the apparatus it was important to see how it would work in practice. For this purpose a rough model of the apparatus was built, for which the frame of an old aperiodic horizontal pendulum was used. This model is illustrated in fig. 3. The pin b is affixed, not at the centre of gravity of the system, but at a point O lying considerably higher, namely at the distance l' from E, in order to increase the pressure on the quartz plate. The mass M was given a cylindrical shape, as shown in the figure.

This undoubtedly is a very rough model of the apparatus devised above, since the whole system does not possess sufficient solidity and stability. Moreover, the bar EG with the weight M may bend somewhat and thus introduce additional oscillations. The apparatus should in practice be constructed as shown in fig. 2, the pin b passing through the centre of gravity of the system. The pressure on the quartz plate will be diminished in this way, but this may be compensated by increasing the weight M, or the sensitiveness of the electrometer.

Suppose the distance EO in the model (fig. 3) is equal to l'. It is evident that in formulæ (8), (10), (13), (14), and (15), M and m must be replaced by $M(l/l_1)$ and $m(l/l_1)$, so that we shall have:

$$P_0 = \frac{l}{l'} Mg \sin \alpha, \tag{8'}$$

$$p = \frac{l}{l'} Mw \cos \alpha, \tag{10'}$$

$$A = \frac{1}{k \frac{l}{l'} M \cos \alpha}, \tag{13'}$$

$$p_1 = mg \sin \alpha \frac{L}{l'}, \tag{14'}$$

$$k = \frac{l'}{L} \cdot \frac{y_1}{mg \sin \alpha}. \tag{15'}$$

Formula (16) remains unaltered.

The model was placed on the platform mentioned above and carefully tested in every way. The conditions of working in Petrograd were very unfavourable owing to the street traffic, so that a number of the testing experiments had to be carried out at night.

The surface of the revolving drum was about 329·9 cm. distant from the optical centre of the objective of the microscope, the focal distance of which was 1·50 cm. The magnification of the optical system was therefore 219, *i.e.*, a displacement of 1 mm. of the image of the string on the registering cylinder corresponded to an actual displacement of 0·0046 mm. of the string itself.

The dimensions and weights of the different parts of the apparatus were as follows :—

$$\alpha = 12{\cdot}6^\circ,\ l' = 24 \text{ cm.},\ l = 58{\cdot}0 \text{ cm.},\ L = 63{\cdot}7 \text{ cm.},\ M = 14{\cdot}04 \text{ kgrm.},$$
$$m = 224{\cdot}8 \text{ grm.}$$

The quartz plate was 5 cm. long, 1 cm. wide, and 0·2 cm. thick. The diameter of the string was between two and three microns, the charge on the plates of the electrometer was +60 and −60 volts. A displacement of 1 mm. of the image of the string on the registering drum was obtained by charging the string to a potential of 0·018 vol.

The regular harmonic oscillations indicated by equation (4) were then communicated to the platform, and the amplitude, a, as well as the period T_p, of these oscillations were determined in the usual way. We shall then have:

$$w = \frac{4\pi^2}{T_p^2} a \sin 2\pi \frac{t}{T_p},$$

and in accordance with the equation (12):

$$y = \frac{1}{A} \cdot \frac{4\pi^2}{T_p^2} a \sin 2\pi \frac{t}{T_p}. \tag{17}$$

In other words, when the platform executes regular harmonic oscillations, the curve y also represents a sinusoid with the same period T_p *and without any difference of phase.*

This result was well confirmed by the experiments, as shown by the curves (figs. 4, 5, and 6), representing in full scale the movements of the platform (lower curves), and the corresponding movements of the string of the electrometer (upper curves). The three curves represent three different periods of oscillations of the platform. The time marks are represented by projections on the lower curves and white lines (dashes) on the upper curves. They do not come opposite each other, which is explained by the parallax of the pen marking the time on the curves of movement of the platform. The same remark may be applied to the corresponding maxima and minima in both curves; but careful measurements of the curves have shown that within the limits of experimental error there is no difference of phase between them. In the following Table the periods of the oscillations of the platform are compared with the corresponding periods of the string of the electrometer. Since the velocity of the registering cylinder was not strictly uniform, errors of some hundredths of a second are to be expected. A length of 4 mm. on the registering drums represents an interval of 1 second.

Table I.

T_p of the platform.	T_p of the electrometer.	ΔT_p.
sec.	sec.	sec.
3·26	3·38	−0·02
3·25	3·29	−0·04
3·25	3·30	−0·05
Mean... 3·253	Mean... 3·290	−0·037
1·81	1·82	−0·01
1·82	1·82	0
Mean... 1·815	Mean... 1·820	−0·005
0·827	0·814	+0·013
0·818	0·835	−0·017
0·854	0·834	+0·020
0·824	0·822	+0·002
Mean... 0·831	Mean ...0·826	+0·005

The Table shows that within the limits of experimental error the period of the platform coincides with that of the electrometer.

Let us now compare the maximum amplitudes of the curves. Let the

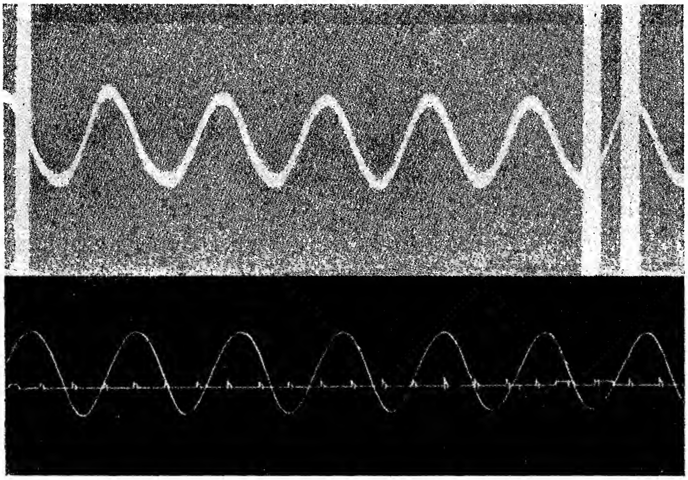

Fig. 4.

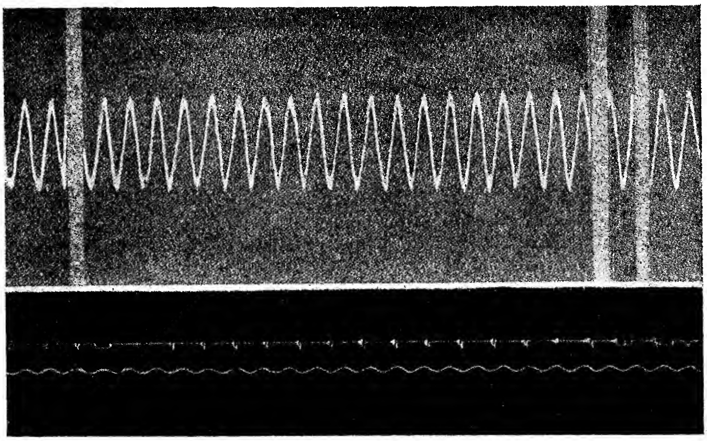

Fig. 5.

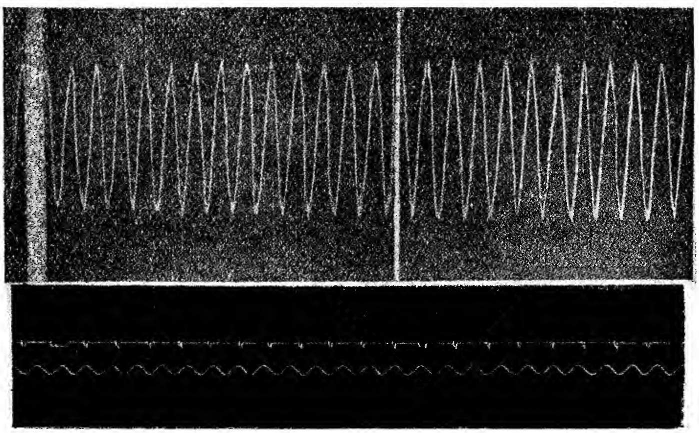

Fig. 6.

maximum amplitude of the curve of the electrometer be y_m. Putting $q = y_m/a$, we get from formula (17)

$$q = \frac{1}{A} \cdot \frac{4\pi^2}{T_p^2}. \tag{18}$$

The amplitudes y_m and a were determined simply by using a glass millimeter scale.

In Table II the values of q are compared with the corresponding periods T_p of the platform. The limits of the amplitudes a corresponding to the values of q are given in a separate column.

Table II.

Periods.	Limits of a.	q.	Mean value of q.
sec.	cm.		
3·253	0·68–0·59	1·07	1·15
		1·29	
	2·48–2·19	1·08	
		1·12	
		1·14	
	1·24–0·03	1·29	
		1·07	
1·815	0·83–0·71	3·86	3·72
		3·86	
		3·83	
	0·49–0·40	3·65	
		3·53	
		3·58	
0·831	0·28–0·23	14·8	16·23
		16·4	
		16·0	
		15·7	
		15·6	
	0·12–0·10	16·7	
		16·5	
		16·1	
		16·8	
		16·3	
	0·24–0·19	15·2	
		15·1	
		15·1	
		15·5	
		15·4	
		15·2	
	0·07–0·05	17·1	
		18·0	
		17·8	
		17·5	
		18·1	

The Table II gives three pairs of corresponding mean values of T_p and q, namely:

$$T_{p_1} = 3{\cdot}253 \text{ sec.} \qquad q_1 = 1{\cdot}15.$$
$$T_{p_2} = 1{\cdot}815 \text{ sec.} \qquad q_2 = 3{\cdot}72.$$
$$T_{p_3} = 0{\cdot}831 \text{ sec.} \qquad q_3 = 16{\cdot}23.$$

Equation (18) shows that q is inversely proportional to the square of the period T_p. If we try to verify this relation we get,

$$\frac{q_2}{q_1} = 3{\cdot}2, \qquad \frac{q_3}{q_1} = 14.$$
$$\frac{T_{p_1}^2}{T_{p_2}^2} = 3{\cdot}2, \qquad \frac{T_{p_1}^2}{T_{p_3}^2} = 15.$$

We see that the experimental results are in good agreement with the theory, which shows that the apparatus really gives the value of the acceleration. The small difference between $q_3/q_1 = 14$ and $T_{p_1}^2/T_{p_3}^2 = 15$ is probably due to the fact that for short periods such as $T_{p_3} = 0{\cdot}831$ sec., the instability of the model already becomes manifest, so that the recorded values of y_m and q are somewhat smaller than in reality.

The value of q now being known, the constant A of the apparatus can be determined from formula (18), which may be written in the following way:—

$$A = \frac{4\pi^2}{qT_p^2}. \tag{19}$$

The mean value of A is 3·3.

Let us now compare this value of A with that which may be obtained from formula (16). On the average the deviation of the electrometer due to the pressure of the additional weight, $m = 224{\cdot}8$ gr., was about 1·11 cm. Since the model used was not provided with a special device for putting on and taking off this additional weight, small accidental charges due to shocks could be expected to appear in the electrometer. This source of error, however, proved to be small, for if we put in formula (16) the value of the deviation y_1 mentioned above, we get

$$A = 3{\cdot}5,$$

which differs very little from $A = 3{\cdot}3$, as given above.

From formula (12) we get for the acceleration, w,

$$w = 3{\cdot}3y.$$

This connection between w and y makes it possible to find out with what accuracy the apparatus allows us to determine the acceleration. In measuring y the possible error we may make, owing to the spreading of the line on the photographic film, is 0·2 mm. Putting $\delta y = 0{\cdot}02$ cm., we get

$$\delta w = 0{\cdot}066 \text{ gal., or,}$$
$$\delta w = 0{\cdot}66 \text{ mm./sec.}$$

This is less than 0·0007 of the acceleration due to gravity.

In conclusion it may be of interest to describe some other experiments carried out by means of this apparatus. The platform was suspended by four flat iron bars of equal thickness. Each of these bars had its own definite period of oscillation, which could be determined directly by using a stopwatch, and was found to be 0·393 sec. In a series of experiments, when small leaden weights were thrown on the platform, or a slight and sharp shock was communicated to it, or one of the bars was struck in a certain way, the electrometer showed a peculiar type of oscillation of very short period, as may be seen from fig. 7, representing these oscillations in their natural size.

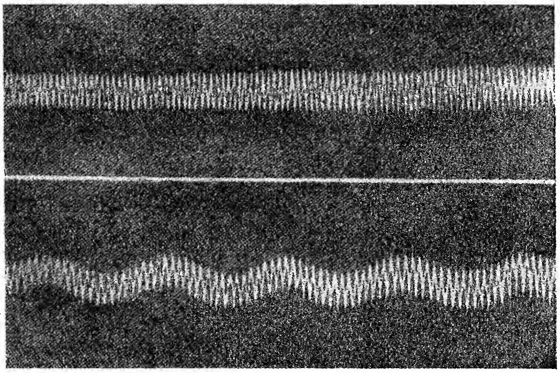

Fig. 7.

The period of these oscillations was always found to be equal to 0·197 sec. At first the origin of this small period was somewhat incomprehensible, but it was recognised afterwards that it was exactly half the period of the vibrations of the bars. It appears that the apparatus may be especially useful for the investigation of very short oscillations.

In further experiments, in order to get rid of the vibrations of the bars, the latter were removed, the apparatus being supported by four brick pillars. In this case, when weights were thrown on to the platform, or sharp shocks communicated to it, two distinct types of oscillations could be detected by the electroscope, viz., oscillations with a period of 0·21 sec., which were due probably to the bending of the rod supporting the mass M of the apparatus, and very short oscillations with a period of the order of 0·05 sec., which may be ascribed to the elastic oscillations of the platform itself.

The apparatus described above can be used for the determination of horizontal accelerations only, but on the same principle an apparatus may be constructed for the determination of vertical accelerations. Fig. 8, representing in principle such an apparatus, is so simple that no further explanations are required.

It can easily be seen that the horizontal accelerations, as determined by means of our apparatus, are influenced by the vertical component, F'', of the acceleration, since the pressure on the quartz plate produced by the mass M changes also with F''. Formulæ (8), (9), and (10) show that, to the normal

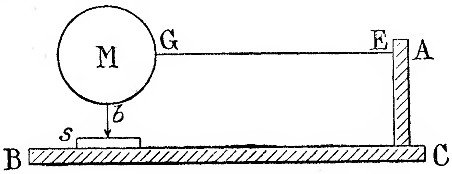

Fig. 8.

pressure, $p = Mw \cos \alpha$, a pressure $MF'' \sin \alpha$ must be added, due to the vertical acceleration F''. Since α is small, the additional pressure forms only a small part of the whole pressure produced on the quartz plate. A correction for this additional pressure may be easily made, since the vertical component of the acceleration, as determined by the apparatus (fig. 8), is not influenced by the horizontal acceleration w.

Summing up, we come to the following conclusions:—

The apparatus described above for the direct determination of accelerations proves to satisfy fully the object for which it was designed. It gives directly, without any appreciable retarding effect, the instantaneous value of the acceleration, however arbitrary the type of motion may be. It does not introduce any oscillations of its own, it has a very small inertia, and does not manifest any fatigue. Its sensitiveness may be regulated as desired.

In the case of short oscillations, the apparatus proves to be particularly convenient, since its sensitiveness grows rapidly with the decrease of the period. The constant of the apparatus may be determined without any difficulty, and, this being done, the acceleration is then obtained directly in C.G.S. units. The apparatus may be constructed in a very compact form and made portable. For the determination of the three components of the acceleration, three sets of apparatus are necessary.

The apparatus may be used also for the investigation of oscillations in different parts of bridges, ships, and buildings of all kinds, caused by shocks, explosions, earth tremors, or by the action of powerful engines. It may be used also in all cases where it is required to determine the instantaneous value of an acceleration, as, for example, in gusts of wind, or in drawing diagrams of pressure in different kinds of steam-engines. It may serve also for the determination of the direction of motion, when this cannot be done by referring to the position of a fixed point, or when there are no near objects for comparison, as, for example, in aeroplanes, airships, submarines, and so on.

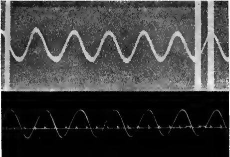

Fig. 4.

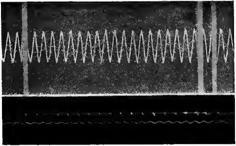

Fig. 5.

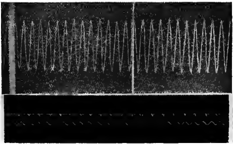

FIG. 6.

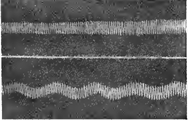

Fig. 7.